Student Name: ____________________

Maths Class: ____________________

James Ruse Agricultural High School

Year 12 Trial HSC Examination 2023

Mathematics Advanced

Reading time: 10 minutes
Working time: 3 hours

General Instructions

- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- In Questions 11-32, show relevant mathematical reasoning and/or calculations.

Total Marks: 100

Section I

- Attempt questions 1-10.
- Answer on the multiple choice answer sheet provided.
- Allow approximately 15mins for this section.

Sections II, III and IV

- Attempt questions 11-32.
- Answer on the space provided on the booklet. These spaces provide guidance for the expected length of response.
- Allow approximately 2hrs 45mins for these three sections.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Additional writing space is provided at the back of each section. If you use this space, clearly indicate which question you are answering.

Section I
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

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1. Which of the following transformations would produce a graph with the same x-intercepts as $y = f(x)$?

   (A) $y = -f(x)$
   (B) $y = f(-x)$
   (C) $y = f(x+1)$
   (D) $y = f(x) + 1$

2. Which term of the G.P. is represented by 64 as the last term

   $$\{\frac{1}{\sqrt{2}}, 1, \sqrt{2}, \ldots\ldots\ldots\ldots\ldots, 64\}?$$

   (A) 6
   (B) 9
   (C) 11
   (D) 14

3. What is the equation of the tangent to the curve $y = \sin x$ at the origin?

   (A) $y = -x$
   (B) $y = \cos x$
   (C) $y = \sin x$
   (D) $y = x$

4. $\log_c(a) + \log_a(b) + \log_b(c)$ is equal to

   (A) $\frac{1}{\log_c(a)} + \frac{1}{\log_a(b)} + \frac{1}{\log_b(c)}$

   (B) $\frac{1}{\log_a(c)} + \frac{1}{\log_b(a)} + \frac{1}{\log_c(b)}$

   (C) $-\frac{1}{\log_c(a)} - \frac{1}{\log_a(b)} - \frac{1}{\log_b(c)}$

   (D) $\frac{1}{\log_a(a)} + \frac{1}{\log_b(b)} + \frac{1}{\log_c(c)}$

5. Which of the following can represent the nth term of a GP?

(i) n^3 (ii) $2^n + 3^n$ (iii) $2^n \times 3^n$ (iv) $3^2 \times 2^n$

(A) iii only

(B) iii, iv only

(C) i, iii, iv only

(D) i, ii, iii and iv

6. Which of the following is a correct expression for $\frac{d}{dx}\text{cosec}\,(x)$

(A) $-\sec(x)\tan(x)$

(B) $-\sec(x)\cot(x)$

(C) $-\text{cosec}(x)\tan(x)$

(D) $-\text{cosec}(x)\cot(x)$

7. Which of the following is a possible solution of $\int \frac{2}{3-4x}dx$

(A) $\frac{1}{2}\ln|4x-3|+2$

(B) $2\ln|4x-3|+2$

(C) $\frac{-1}{2}\ln|4x-3|+2$

(D) $-2\ln|4\text{x}-3|+2$

8. Which of the following conditions over the interval [a,b] will result in the trapezoidal rule overestimating the value of the integral $\int_a^b f(x)dx$

(A) $f(x)$ is increasing

(B) $f(x)$ is decreasing

(C) $f(x)$ is concave up

(D) $f(x)$ is concave down

9. Using the identity $\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$ from the Reference Sheet, or otherwise, identify which expression is equal to $\int \sin^2 3x \, dx$?

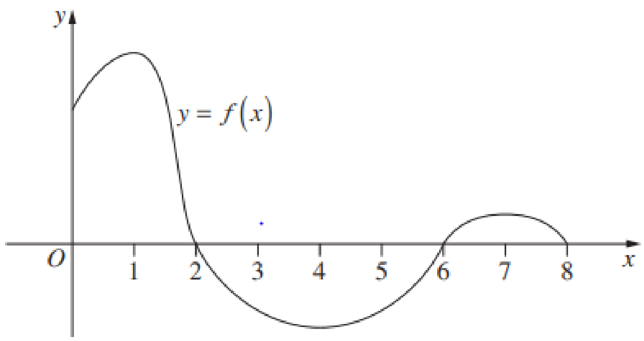
(A) $\frac{1}{2}\left(x - \frac{1}{3}\cos 6x\right) + C$

(B) $\frac{1}{2}\left(x + \frac{1}{3}\cos 6x\right) + C$

(C) $\frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + C$

(D) $\frac{1}{2}\left(x + \frac{1}{6}\sin 6x\right) + C$

10. The graph of $y = f(x)$ has been drawn to scale for $0 \leq x \leq 8$.



Which of the following integrals has the greatest value?

(A) $\int_0^1 f(x)\,dx$

(B) $\int_0^2 f(x)\,dx$

(C) $\int_0^7 f(x)\,dx$

(D) $\int_0^8 f(x)\,dx$

End Section I

Section I **Student Number: ______________________**

Attempt Questions 1 – 10

Allow approximately 15 minutes for this section

Use the multiple-choice answer sheet below to record your answers to Question 1 – 10.

Select the alternative: A, B, C or D that best answers the question.

Colour in the response oval completely.

Sample:

2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, draw a cross through the incorrect answer and colour in the new answer

ie A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word "*correct*" and draw an arrow as follows:

correct

A ● B ~~●~~ C ○ D ○

Year 12 Trial 2023

Mathematics Advanced Trial HSC Examination

Multiple Choice Answer Sheet

Completely colour in the response oval representing the most correct answer.

1	A	○	B	○	C	○	D	○
2	A	○	B	○	C	○	D	○
3	A	○	B	○	C	○	D	○
4	A	○	B	○	C	○	D	○
5	A	○	B	○	C	○	D	○
6	A	○	B	○	C	○	D	○
7	A	○	B	○	C	○	D	○
8	A	○	B	○	C	○	D	○
9	A	○	B	○	C	○	D	○
10	A	○	B	○	C	○	D	○

Mark: /10

Section II (30 marks) Student Number:______________

Answer in the space provided.

11. Justify whether $f(x) = (x-1)^2$ is an even function. **2**

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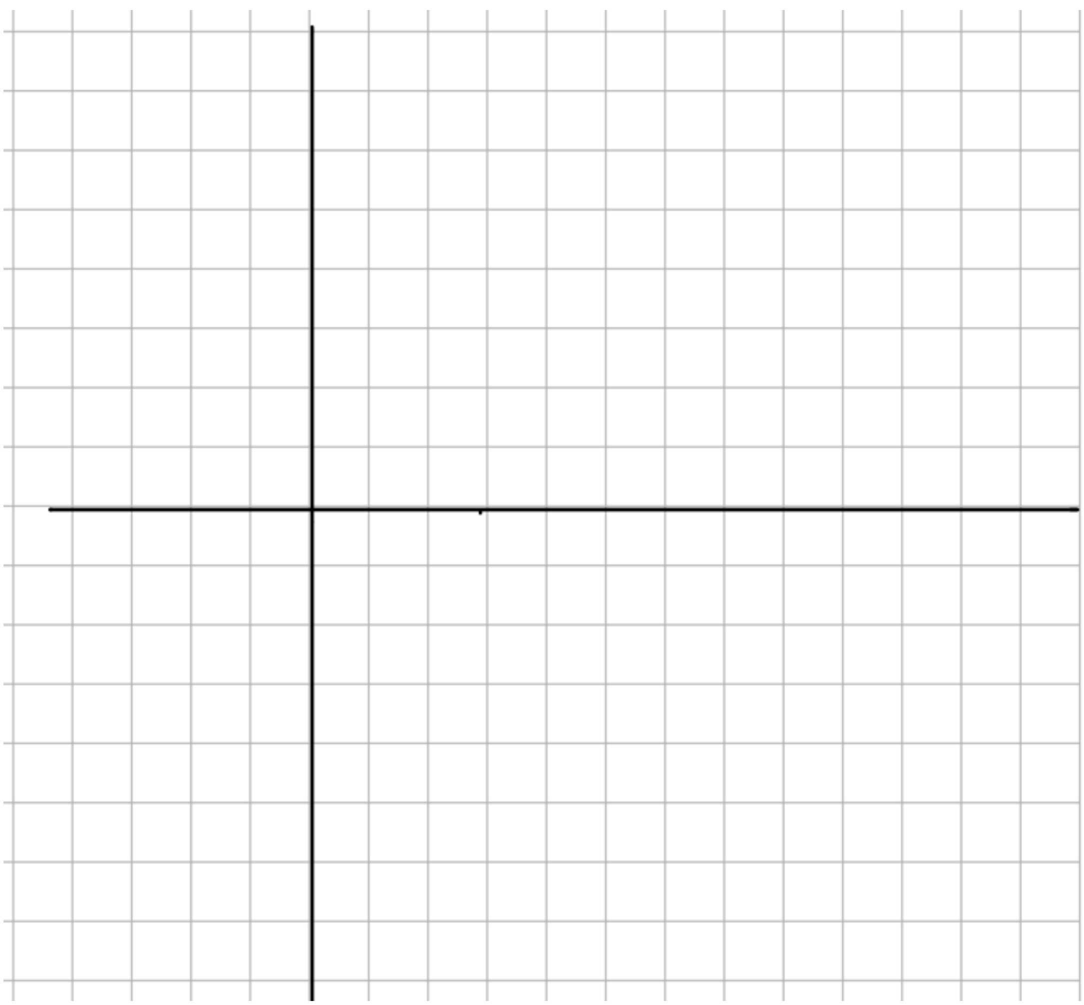
12. Sketch $y = 3\log_{10}(x-2)$ showing all important features. **3**



13. Solve $| 4x - 8 | = 3 | x + 2 |$ **2**

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14. The point $P(-1,\ 3)$ lies on the curve with equation $y = f(x)$. **3**
State the coordinates of the image of point P (called P′) after the following transformations have taken place:

i. $y = f(x + 3) + 3$

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ii. $y = 2f(x) + 3$

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iii. $y = f(2x + 3)$

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15. Show whether $g(x) = \frac{x-1}{x+1}$ is monotonic increasing or decreasing. **2**

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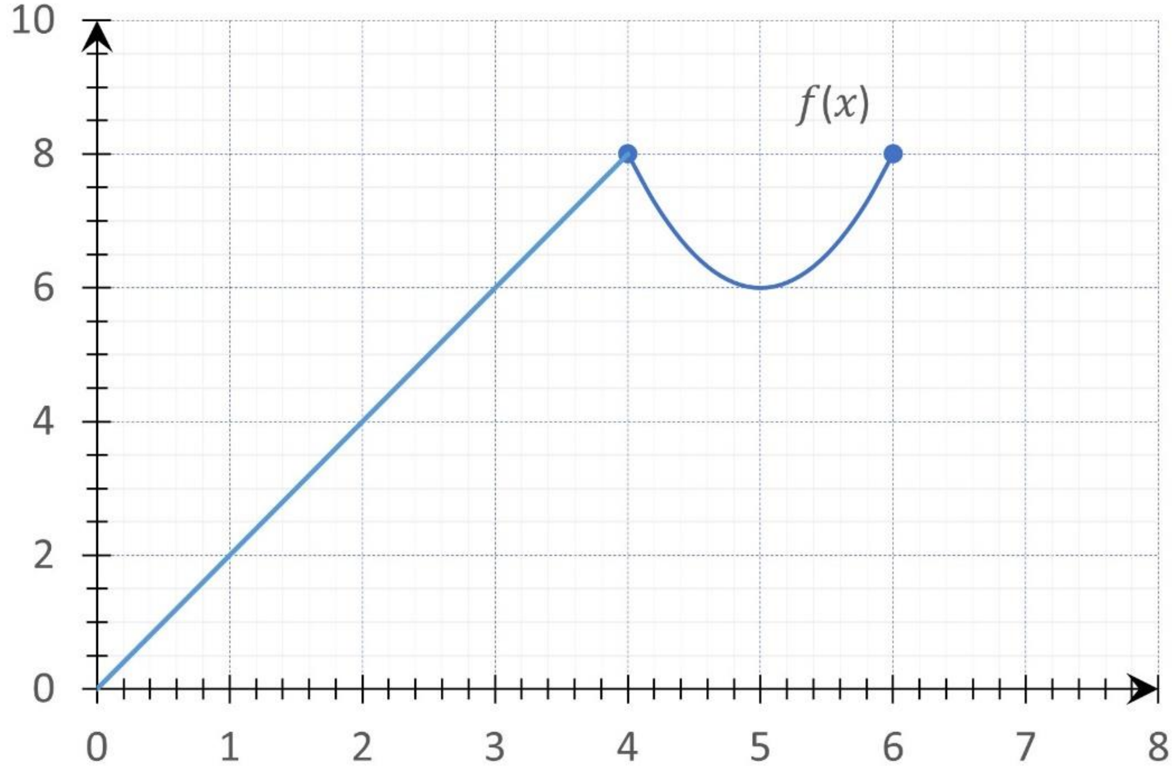
16. Given the graph of $f(x)$ as shown in the diagram below, sketch on the same axes $g(x)$ and $h(x)$. Label your graphs clearly. **3**

i. $g(x) = f(2x)$.

ii. $h(x) = 0.5f(x)$



17. For the random variable X, it is known that E(X) =6 and Var(X) =2 . **1**
Write down E($3X - 1$) for the new distribution $3X - 1$

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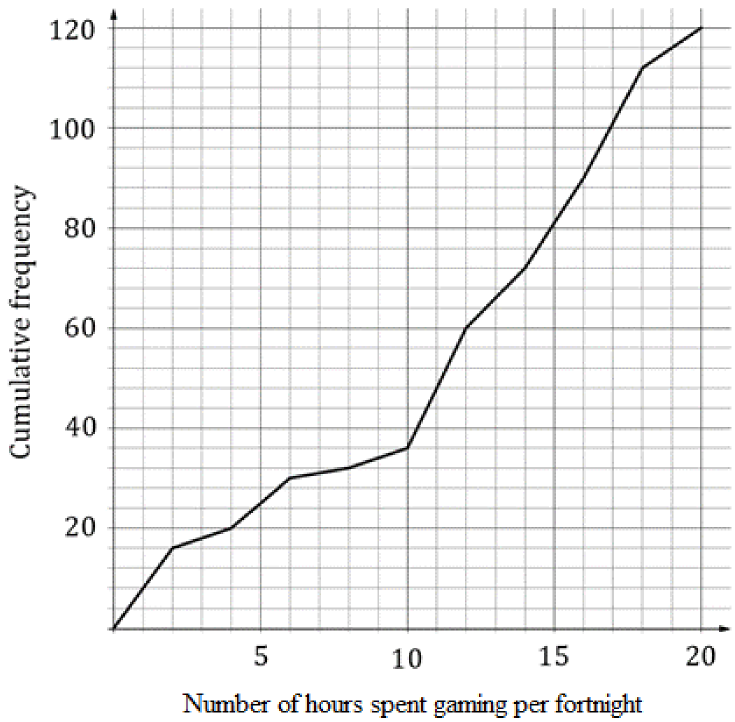
18. The following cumulative frequency curve shows the number of hours spent gaming per fortnight by 120 high school students.



(a) Find the Median number of hours spent gaming. **1**

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(b) Find the Interquartile range. **2**

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(c) Calculate the percentage of students that spent less than 17 hours gaming per fortnight. **1**

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(d) Calculate the maximum number of hours that can be spent gaming per fortnight and not be considered an outlier. **2**

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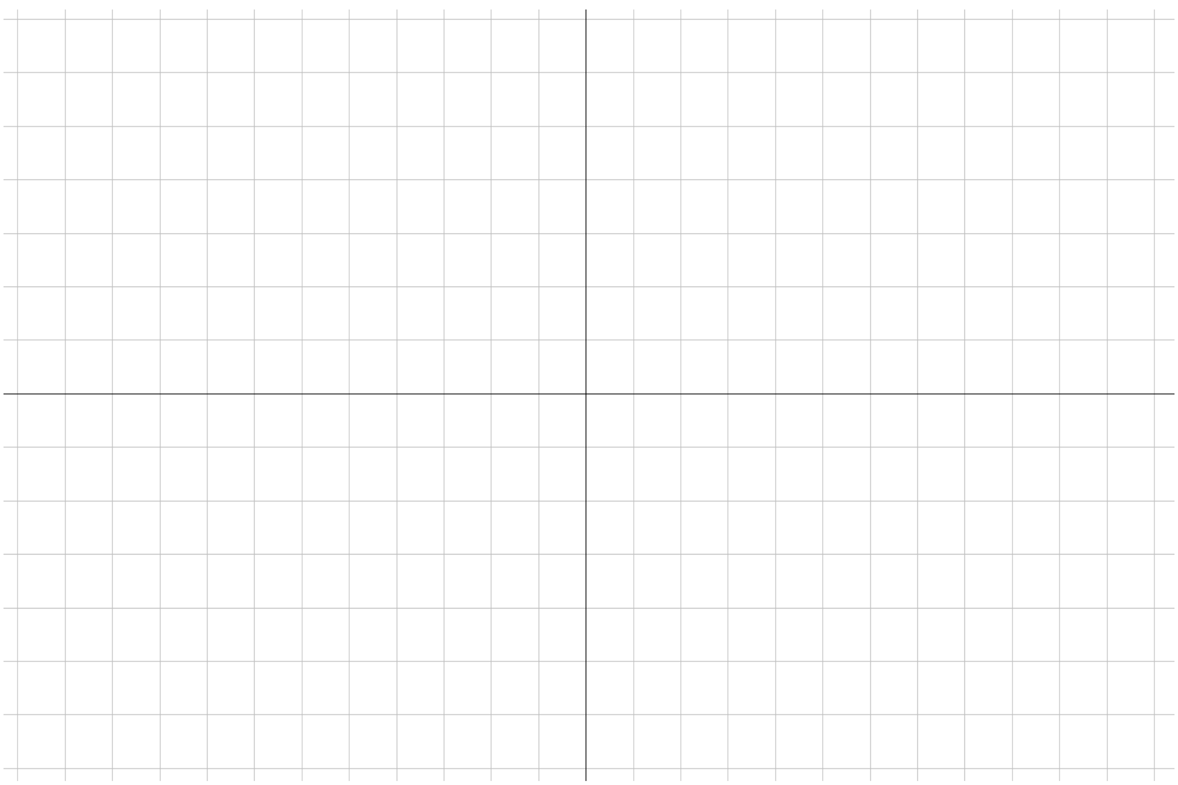
19. Let $f(x) = \left(\frac{1}{2}x - 2\right)^3 + 2$

(a) Show the equation of the tangent to $f(x)$ at the point where $x = 6$ is $y = \frac{3}{2}x - 6$ **2**

(b) On the Graph paper below, graph the functions $f(x)$ and the tangent of $f(x)$ at $x = 6$, showing all important features **3**



(c) Calculate the area above the x-axis and enclosed between $f(x)$ and the tangent, where $0 < x < 6$.

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End Section II

Section III (30 marks) **Student Number:____________**

Answer in the space provided.

20. Find all solutions for $\cos x + \sin^2 x = \frac{5}{4}$, $-\pi \le x \le \pi$ **3**

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21. Find the derivatives of the following :

a. $y = \frac{x}{x^2-5}$ **2**

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b. $y = x^2\sqrt{4x-1}$ **2**

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22. Find $\int 7x^2 \sin(2x^3 - 5)\, dx$ **3**

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23. A function has equation $y = e^{2x}$. Find the equation of the normal to the curve at the point where $x = 1$. **2**

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24. A continuous random variable has the probability density function given by:

$$f(x) = \begin{cases} ax^3, & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

(a) Find the value of a **2**

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(b) Sketch the graph of $f(x)$ **3**

(c) Calculate

$$P(X < \frac{2}{3} \mid X > \frac{1}{3})$$

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25. The population L in thousands of mountain lions, who are predators, and the population D in thousands of deer, who are the prey, in a particular continuously monitored region can be modeled by the following two periodic functions.

$$L = 9 + 6\sin\frac{\pi}{4}(t-1)$$

$$D = 18 + 9\cos\frac{\pi}{4}t$$

Where t is the time in years after the Tracking with Technology started, L and D are the populations (in thousands) of lions and deer respectively.

(a) How much time passes between successive maximum lions' populations (the period)? **2**

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(b) Determine the initial populations of lions and deer respectively. **2**

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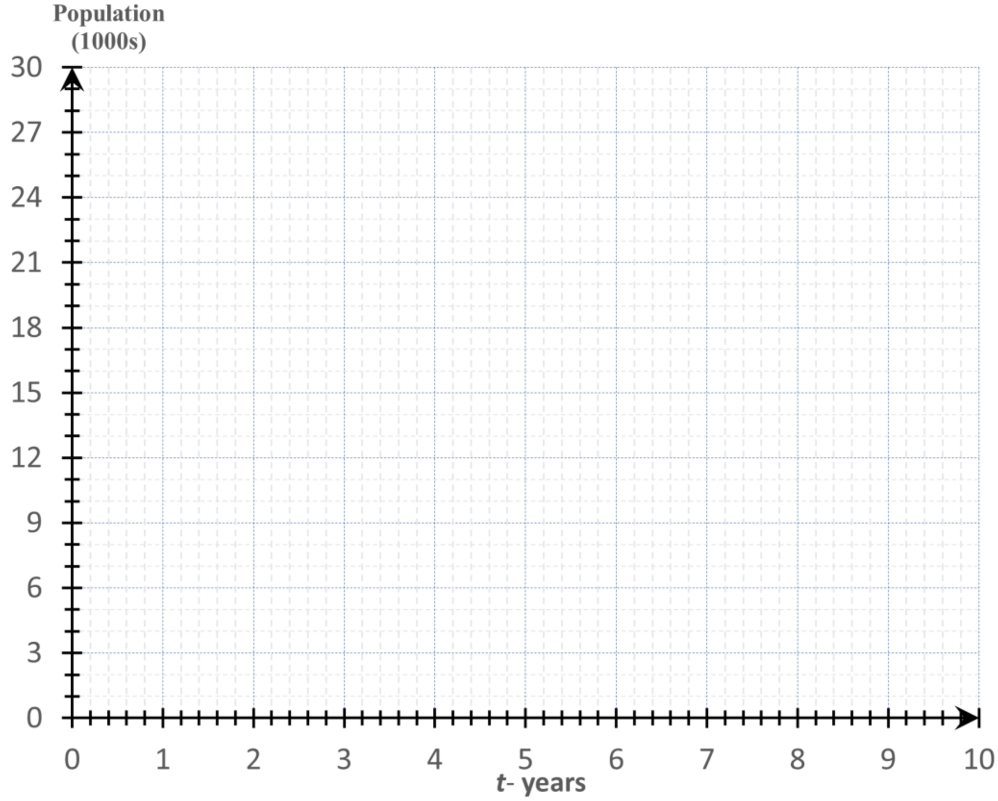
(c) Sketch the graphs of the two functions (the Lion and Deer functions) on the same axes in the domain $0 \leqslant t \leqslant 10$ **2**



(d) Over which time period, $0 \leqslant t \leqslant 10$, was the deer population increasing while the lion population decreased? **1**

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26. Solve $\ln x - \frac{6}{\ln x} = 5$ **2**

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27. If $x^2 = (2a - x)(2b - x)$ show that $\frac{1}{a}$, $\frac{1}{x}$, $\frac{1}{b}$ are in arithmetic sequence. **3**

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End Section III

Section IV (30 marks) **Student Number:_____________**

Answer in the space provided.

28. Katie has deliberately designed a biased six sided die, with the following probability distribution for X, the number on the uppermost face when the die is rolled.

x	**1**	**2**	**3**	**4**	**5**	**6**
$P(X = x)$	$\frac{1-\theta}{6}$	$\frac{1-\theta}{6}$	$\frac{1-\theta}{6}$	$\frac{1+\theta}{6}$	$\frac{1+\theta}{6}$	$\frac{1+\theta}{6}$

(a) What values of θ are feasible in order for $P(X)$ to be a probability function? **2**

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(b) Find $P(0 \leq X \leq 4)$ **1**

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(c) Find the probability of rolling an even number? **1**

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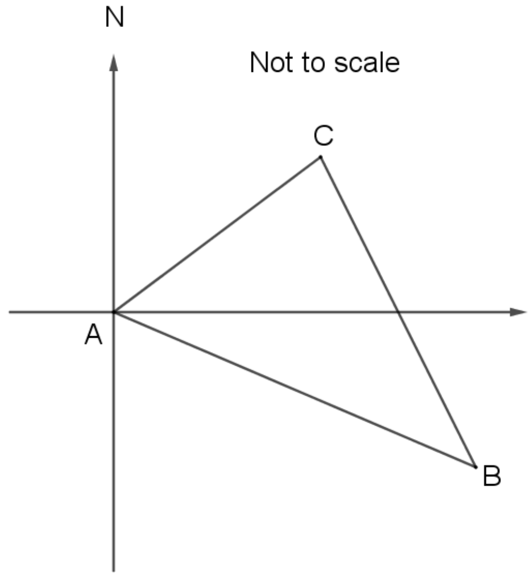
29. A student on a bike left point A and walked 1.5km on a bearing of 120^0T to a point B.
He then turned and walked 10km on a bearing of 330^0T to a point C.



(a) Label all features on the diagram above. **2**

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(b) Calculate the distance from point C to point A. (give your answer to 2 decimal places) **2**

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(c) What is the bearing of A from C ? (Give your answer to the nearest degree) **2**

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30. A particle is moving in a straight line. Initially, it is travelling to the left at 1cm/min. Its acceleration is given by:

$$a = \pi \cos\ (\pi t) + \pi \sin\ (\pi t)$$

for $0 \leq t \leq 2$ where time and displacement are measure in minutes and cm respectively.

(a) Find when the particle first changes its direction. **2**

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(b) Find the total distance travelled in the first half a minute. **3**

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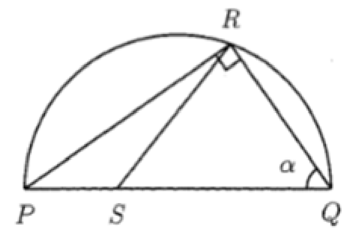
31. ΔPQR is a right-angled triangle inscribed in a semi-circle. R is a variable point on the circumference. The point S lies on PQ such that $SQ = kQR$ where k is a positive constant. If $PQ = d\ cm$ and $\angle PQR = \alpha$ radians:



(a) Show that the area of ΔSQR is $A = \frac{1}{2}kd^2cos^2\alpha sin\alpha$. **3**

(b) Show that $\frac{dA}{d\alpha} = \frac{1}{2}kd^2(3cos^3\alpha - 2cos\alpha)$. **2**

(c) Find the greatest possible area of ΔSQR in terms of k and d. **3**

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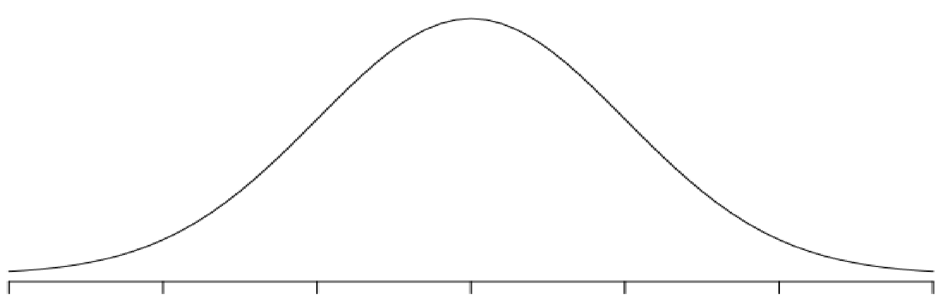
32. An airline company operates regular flights between cities A and B. The flight time X is normally distributed with a mean of 80 minutes and standard deviation σ minutes. Also, 2.5% of the flights take longer than 96 minutes to arrive at the destination.

(a) Label the normal distribution curve below with the given information. **1**



(b) Find the value of σ. **1**

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(c) What percentage of flights take longer than 90 minutes? **2**

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(d) In order to attract customers, the management of the airline company decides to refund the fares if a flight takes longer than $(80 + t)$ minutes. The company wants to keep 99.5% of the fares. Calculate the value of t. **3**

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End of Section IV and END OF EXAMINATION

Student Name: ________________________

Maths Class: ________________________

James Ruse Agricultural High School

Year 12 Trial HSC Examination 2023

Mathematics Advanced

Reading time: 10 minutes
Working time: 3 hours

General Instructions

- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11-32, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I

- Attempt questions 1-10
- Answer on the multiple choice answer sheet provided
- Allow approximately 15mins for this section

Sections II, III and IV

- Attempt questions 11-32
- Answer on the space provided on the booklet. These spaces provide guidance for the expected length of response.
- Allow approximately 2hrs 45mins for these three sections.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Additional writing space is provided at the back of each section. If you use this space, clearly indicate which question you are answering.

Section I
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

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1. Which of the following transformations would produce a graph with the same x-intercepts as $y = f(x)$?

   (A) $y = -f(x)$ no change
   (B) $y = f(-x)$ $x \to -x$
   (C) $y = f(x+1)$ $x \to x+1$
   (D) $y = f(x) + 1$ x moves

2. Which term of the G.P. is represented by (64 as) the last term of the set
   $\{\frac{1}{\sqrt{2}}, 1, \sqrt{2}, \ldots \ldots \ldots \ldots \ldots, 64\}$?

   (A) 6
   (B) 9
   (C) 11
   (D) 14

3. What is the equation of the tangent to the curve $y = \sin x$ at the origin?

   (A) $y = -x$
   (B) $y = \cos x$
   (C) $y = \sin x$
   (D) $y = x$

4. $\log_c(a) + \log_a(b) + \log_b(c)$ is equal to

   (A) $\frac{1}{\log_c(a)} + \frac{1}{\log_a(b)} + \frac{1}{\log_b(c)}$
   (B) $\frac{1}{\log_a(c)} + \frac{1}{\log_b(a)} + \frac{1}{\log_c(b)}$
   (C) $-\frac{1}{\log_c(a)} - \frac{1}{\log_a(b)} - \frac{1}{\log_b(c)}$
   (D) $\frac{1}{\log_a(a)} + \frac{1}{\log_b(b)} + \frac{1}{\log_c(c)}$

5. Which of the following can represent the nth term of a GP?

(i) n^3 (ii) $2^n + 3^n$ (iii) $2^n \text{x}\, 3^n$ (iv) $3^2 . 2^n$

(A) iii only
(B) iii, iv only
(C) i, iii, iv only
(D) i, ii, iii and iv

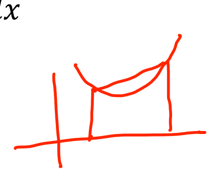
6. Which of the following is a correct expression for $\frac{d}{dx}\text{cosec}\,(x)$

(A) $-\sec(x)\tan(x)$
(B) $-\sec(x)\cot(x)$
(C) $-\text{cosec}(x)\tan(x)$
(D) $-\text{cosec}(x)\cot(x)$

7. Which of the following is a possible solution of $\int \frac{2}{3-4x}dx$

(A) $\frac{1}{2}\ln|4x-3| + 2$
(B) $2\ln|4x-3| + 2$
(C) $\frac{-1}{2}\ln|4x-3| + 2$
(D) $-2\ln|4x-3| + 2$

8. Which of the following conditions over the interval [a,b] will result in the trapezoidal rule overestimating the value of the integral $\int_a^b f(x)dx$

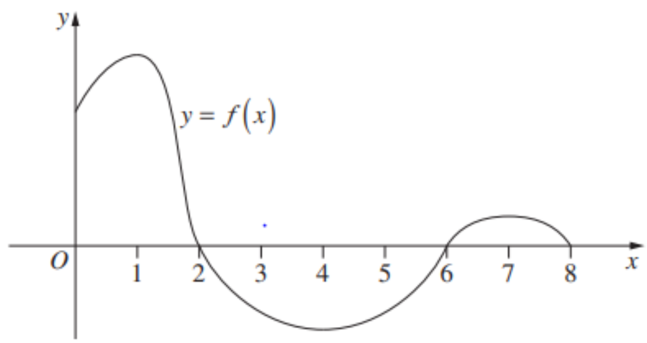


(A) $f(x)$ is increasing
(B) $f(x)$ is decreasing
(C) $f(x)$ is concave up
(D) $f(x)$ is concave down

9. Using the identity $\sin^2 nx = \frac{1}{2}(1 + \cos 2nx)$ from the Reference Sheet, or otherwise, identify which expression is equal to $\int \sin^2 3\,x\,dx$?

(A) $\frac{1}{2}\left(x - \frac{1}{3}\cos 6x\right) + C$
(B) $\frac{1}{2}\left(x + \frac{1}{3}\cos 6x\right) + C$
(C) $\frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + C$
(D) $\frac{1}{2}\left(x + \frac{1}{6}\sin 6x\right) + C$

10. The graph of $y = f(x)$ has been drawn to scale for $0 \le x \le 8$.



Which of the following integrals has the greatest value?

(A) $\int_0^1 f(x)\,dx$
(B) $\int_0^2 f(x)\,dx$
(C) $\int_0^7 f(x)\,dx$
(D) $\int_0^8 f(x)\,dx$

End Section I

Section I **Student Number:** ____________________

Attempt Questions 1 – 10

Allow approximately 15 minutes for this section

Use the multiple-choice answer sheet below to record your answers to Question 1 – 10.

Select the alternative: A, B, C or D that best answers the question.

Colour in the response oval completely.

Sample:

2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, draw a cross through the incorrect answer and colour in the new answer

ie A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word "*correct*" and draw an arrow as follows:

correct

A ● B ⊗ C ○ D ○

Year 12 Trial 2023

Mathematics Advanced Trial HSC Examination

Multiple Choice Answer Sheet

Completely colour in the response oval representing the most correct answer.

1	A	●	B	○	C	○	D	○
2	A	○	B	○	C	○	D	●
3	A	○	B	○	C	○	D	●
4	A	○	B	●	C	○	D	○
5	A	○	B	●	C	○	D	○
6	A	○	B	○	C	○	D	●
7	A	○	B	○	C	●	D	○
8	A	○	B	○	C	●	D	○
9	A	○	B	○	C	●	D	○
10	A	○	B	●	C	○	D	○

Mark: /10

Section II (30 marks) Student Number:______________

Answer in the space provided.

11. Justify whether $f(x) = (x-1)^2$ is an even function. **2**

even if $f(x) = f(-x)$

$f(x) = (x-1)^2 = x^2 - 2x + 1$

$f(-x) = (-x-1)^2$

$= x^2 + 2x + 1$

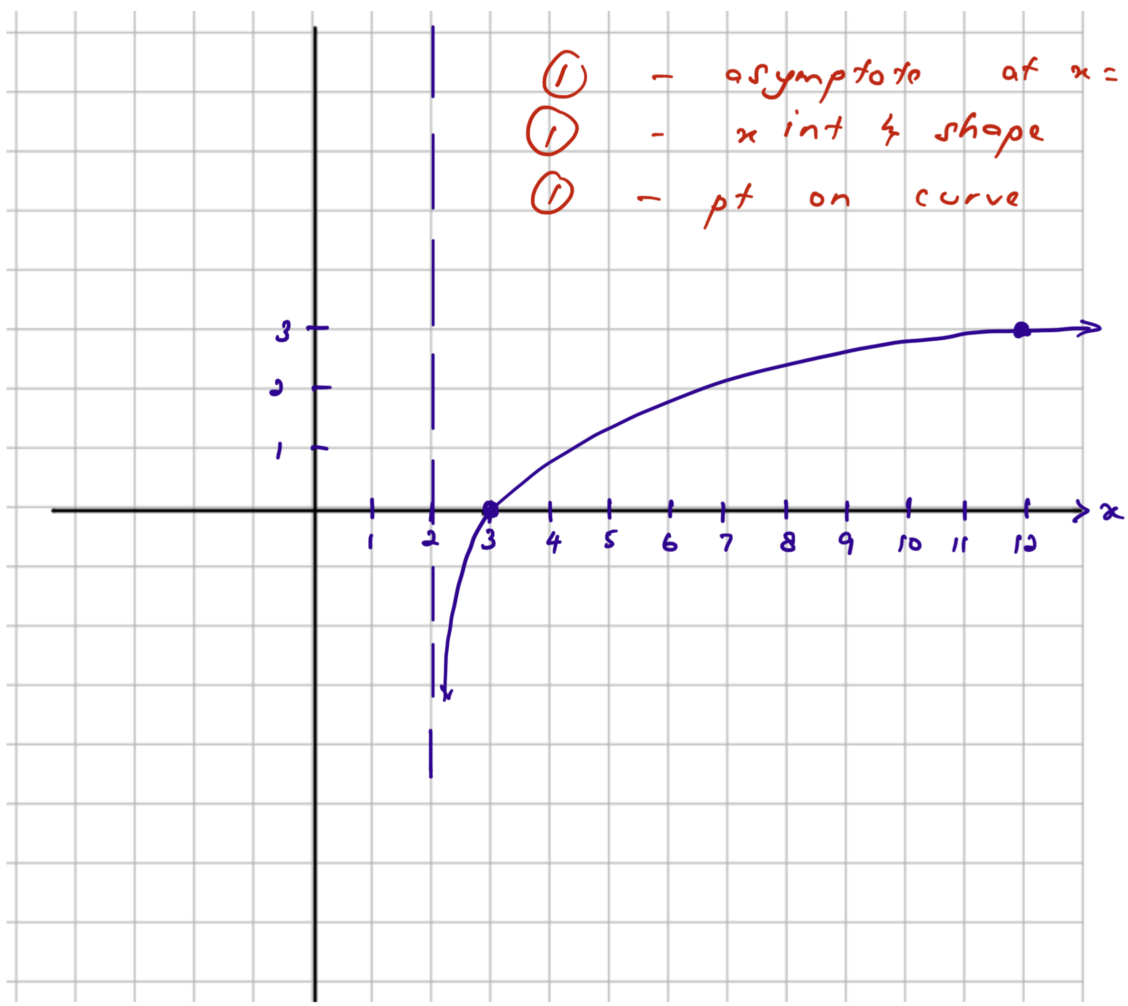
show — ①

$f(x) \neq f(-x)$

∴ $f(x)$ is not an even func

state — ①

12. Sketch $y = 3\log_{10}(x-2)$ showing all important features. **3**



Ext T3

Dhruv

13. Solve $|4x-8| = 3|x+2|$ **2**

Method 1

$4x - 8 = 3(x+2)$

$4x - 8 = 3x + 6$

$x = 14$ — (1)

$4x - 8 = -3(x+2)$

$4x - 8 = -3x - 6$

$7x = 2$

$x = \frac{2}{7}$ — (1)

Method 2 Square both sides

$16x^2 - 64x + 64 = 9(x^2 + 4x + 4)$

$16x^2 - 64x + 64 = 9x^2 + 36x + 36$

$7x^2 - 100x + 28 = 0$

$x = \dfrac{-(-100) \pm \sqrt{(-100^2) - 4(7)(28)}}{2(7)}$

$x = \frac{2}{7}$ or 14

14. The point $P(-1,\ 3)$ lies on the curve with equation $y = f(x)$. **3**
State the coordinates of the image of point P (called P′) after the following transformations have taken place:

i. $y = f(x+3) + 3$

x left 3 $\therefore P' = (-1-3,\ 3+3)$

y up 3 $= (-4, 6)$ — (1)

ii. $y = 2f(x) + 3$

$y \Rightarrow \times 2 + 3$ $P' = (-1,\ 2 \times 3 + 3)$

$= (-1,\ 9)$ — (1)

iii. $y = f(2x+3)$

$f\left(2\left(x + \frac{3}{2}\right)\right)$

$x \Rightarrow$ compressed ie $\times \frac{1}{2}$ $P' = \left(-\frac{1}{2} - \frac{3}{2},\ 3\right)$

then shift left by $\frac{3}{2}$ $= (-2, 3)$ — (1)

15. Show whether $g(x) = \frac{x-1}{x+1}$ is monotonic increasing or decreasing. **2**

M1

$u = x - 1$, $u' = 1$

$v = x + 1$, $v' = 1$

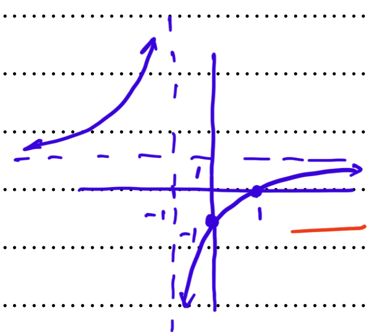
$g'(x) = \dfrac{(x+1)\cdot 1 - (x-1)\cdot 1}{(x+1)^2}$

show (1) — $= \dfrac{2}{(x+1)^2}$

$(x+1)^2 > 0$ then $f'(x) > 0$

state (1) — thus $f(x)$ is monotonic increasing

M2



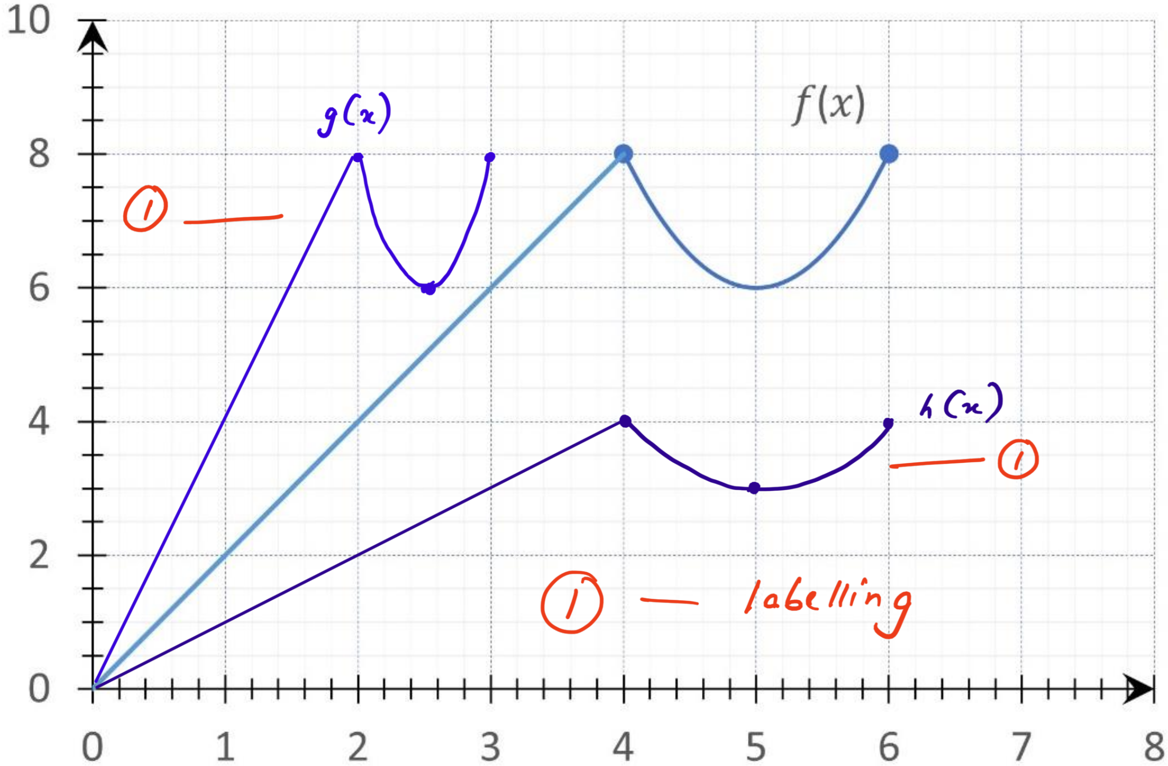
— (1)

then state (1)

16. Given the graph of $f(x)$ as shown in the diagram below, sketch on the same axes $g(x)$ and $h(x)$. Label your graphs clearly. **3**

i. $g(x) = f(2x)$. horizontal dilation $x \to \times \frac{1}{2}$

ii. $h(x) = 0.5f(x)$ vertical dilation $y \to \times \frac{1}{2}$



17. For the random variable X, it is known that $\mathrm{E}(X) = 6$ and $\mathrm{Var}(X) = 2$. **1**
Write down $\mathrm{E}(3X - 1)$ for the new distribution $3X - 1$

$E(x) = 6$

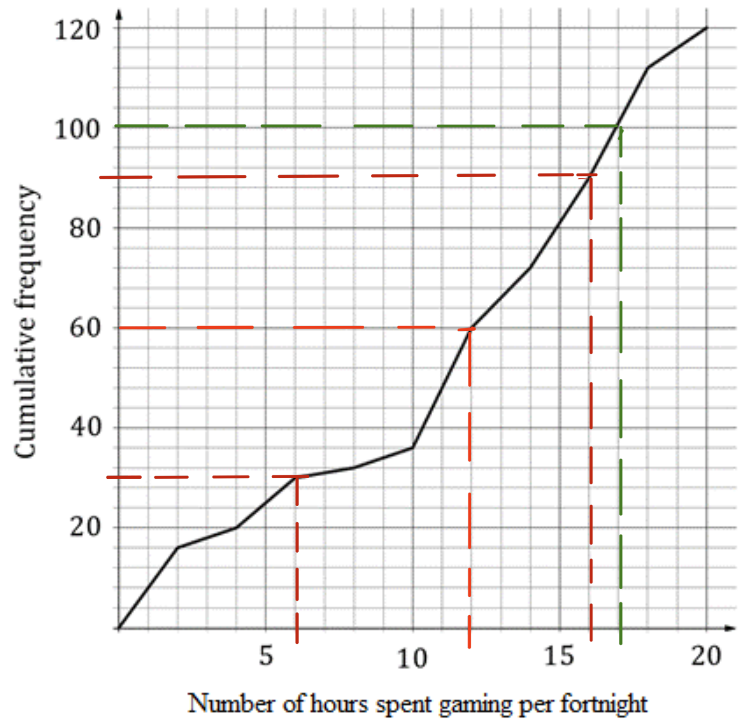
$E(3x - 1) = 3(E(x)) + (-1)$

$= 3(6) - 1$

$= 17$ — ①

18. The following cumulative frequency curve shows the number of hours spent gaming per fortnight by 120 high school students.

25% = 30
50% = 60
75% = 90



(a) Find the Median number of hours spent gaming. **1**

12 hours — (1)

(b) Find the Interquartile range. **2**

16 − 6 — (1)

= 10 — (1)

(c) Calculate the percentage of students that spent less than 17 hours gaming per fortnight. **1**

$\frac{100}{120} \times 100\% = 83.\dot{3}\%$ — (1)

(d) Calculate the maximum number of hours that can be spent gaming per fortnight and not be considered an outlier. **2**

Max no of hrs = ?

Q3 + 1.5 IQR (from reference sheet)

16 + 15

= 31 ie 31 is an outlier — (1)

Not to be an outlier is < 31 — (1)

19. Let $f(x) = \left(\frac{1}{2}x - 2\right)^3 + 2$

(a) Show the equation of the tangent to $f(x)$ at the point where $x = 6$ is $y = \frac{3}{2}x - 6$ **2**

$f'(x) = 3\left(\frac{1}{2}x - 2\right)^2 \cdot \frac{1}{2}$

$= \frac{3}{2}\left(\frac{1}{2}x - 2\right)^2$

$f'(6) = \frac{3}{2}\left(\frac{1}{2}(6) - 2\right)^2$

$= 3/2$ — (1)

when $x = 6$, $y = 3$

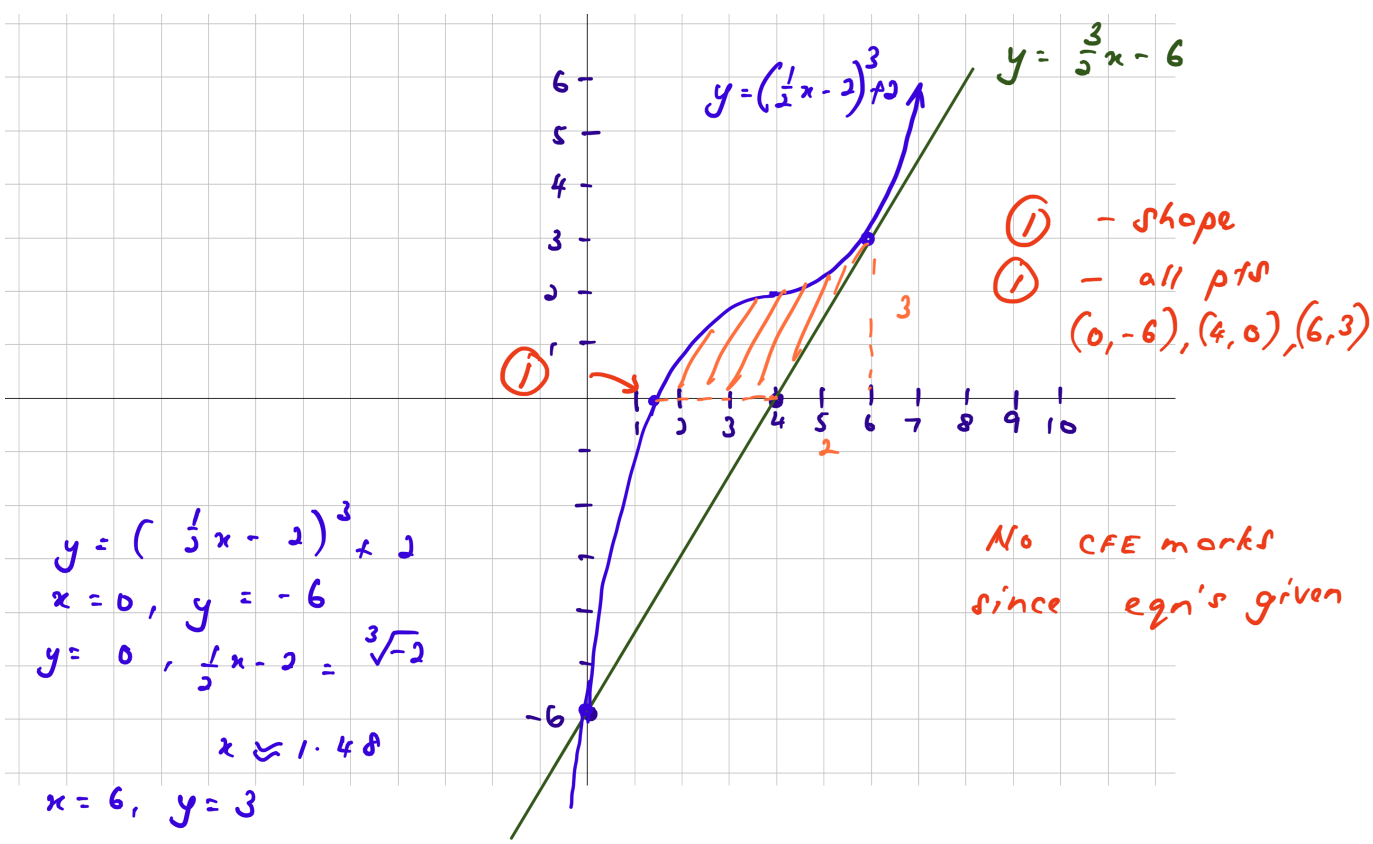
$\therefore$ Eqn of tangent $\Rightarrow$ $y - 3 = \frac{3}{2}(x - 6)$

$y = \frac{3}{2}x - 9 + 3$

$y = \frac{3}{2}x - 6$ (shown) — (1)

Poorly done
Yr 8 content
given a pt & gradient

(b) On the Graph paper below, graph the functions $f(x)$ and the tangent of $f(x)$ at $x = 6$, showing all important features **3**



(c) Calculate the area above the x-axis and enclosed between $f(x)$ and the tangent, where $0 < x < 6$.

3

$= \int_{1.48}^{6} \left(\frac{1}{2}x - 2\right)^3 + 2 \; - \text{A of } \Delta$ — ①

$= \left[\frac{\left(\frac{1}{2}x - 2\right)^4}{4 \cdot \frac{1}{2}} + 2x\right]_{1.48}^{6} - \frac{1}{2} \times 3 \times 2$

— ① integrating correctly

$= 5.279\ldots$

$= 5.3 \text{ (1 d.p)}$ — ①

End Section II

Section III (~~30~~ 32 marks) 32.

Answer in the space provided.

Student Number:____________

20. Find all solutions for $\cos x + \sin^2 x = \frac{5}{4}$, $-\pi \le x \le \pi$ **3**

$\cos x + (1 - \cos^2 x) = \frac{5}{4}$ 1 mark

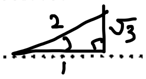
$\cos^2 x - \cos x + \frac{1}{4} = 0$

$4\cos^2 x - 4\cos x + 1 = 0$

$(2\cos x - 1)^2 = 0$ 1 mark

$2\cos x - 1 = 0$

$2\cos x = 1$



$\cos x = \frac{1}{2}$

$\therefore\ x = \frac{\pi}{3}$ or $x = -\frac{\pi}{3}$ 1 mark

21. Find the derivatives of the following :

a. $y = \frac{x}{x^2-5}$ **2**

$\frac{dy}{dx} = \frac{(x^2-5) - x(2x)}{(x^2-5)^2}$ 1 mark for using quotient rule formula correctly. $u = x$ $u' = 1$ $v = x^2 - 5$ $v' = 2x$

$= \frac{x^2 - 5 - 2x^2}{(x^2-5)^2}$

$= \frac{-x^2 - 5}{(x^2-5)^2}$ 1 mark for correct answer.

b. $y = x^2\sqrt{4x-1}$ **2**

$\frac{dy}{dx} = (4x-1)^{\frac{1}{2}}\, 2x + x^2\left(\frac{2}{(4x-1)^{1/2}}\right)$ 1 mark for substituting into Product Rule formula

$u = x^2$ $u' = 2x$

$v = (4x-1)^{\frac{1}{2}}$

$v' = \frac{1}{2}(4x-1)^{-\frac{1}{2}} \cdot 4$

$= 2(4x-1)^{-\frac{1}{2}}$

$= \frac{2}{(4x-1)^{1/2}}$

$= \frac{2x(4x-1) + 2x^2}{(4x-1)^{\frac{1}{2}}}$

$= \frac{8x^2 - 2x + 2x^2}{(4x-1)^{\frac{1}{2}}}$

$= \frac{10x^2 - 2x}{(4x-1)^{\frac{1}{2}}}$ 1 mark for answer

22. Find $\int 7x^2 \sin(2x^3 - 5)\,dx$ 3

$= \frac{7}{6}\int 6x^2 \sin(2x^3-5)\,dx$

1 mark for $\cos(2x^3-5)$

$= -\frac{7}{6}\cos(2x^3-5) + c$

1 mark for $\frac{-7}{6}$

1 mark for $+c$ if most of rest is correct.

23. A function has equation $y = e^{2x}$. Find the equation of the normal to the curve at the point where $x = 1$. 2

$y = e^{2x}$

$y' = 2e^{2x}$

When $x = 1$, $y = e^2$

$y' = 2e^2$

Since the gradient of tangent is $2e^2$ then gradient of normal is $\frac{-1}{2e^2}$

1 mark for gradient of normal

Equation of Normal is: $y - e^2 = \frac{-1}{2e^2}(x-1)$

$2e^2 y - 2e^2 \cdot e^2 = -x + 1$

$x + 2e^2 y - 2e^4 - 1 = 0$

OR $y = \frac{-1}{2e^2}(x-1) + e^2$

$= \frac{-x+1}{2e^2} + \frac{2e^4}{2e^2}$

$= \frac{-x + 2e^4 + 1}{2e^2}$

1 mark for equation in either format.

24. A continuous random variable has the probability density function given by:

$$f(x) = \begin{cases} ax^3, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

(a) Find the value of a **2**

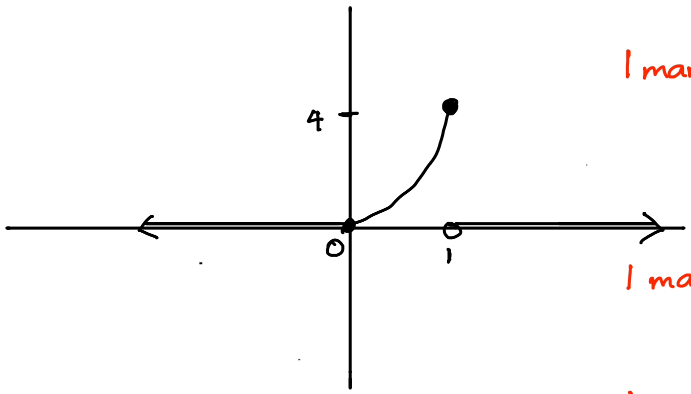
$\int_0^1 ax^3 = 1$

$\left[\frac{ax^4}{4}\right]_0^1 = 1$ 1 mark for correct integration and =1

$\frac{a}{4} - 0 = 1$

$a = 4$ 1 mark for answer.

(b) Sketch the graph of $f(x)$ **3**



1 mark for ax^3 between 0 and 1 on x-axis and 0 to 4 on y-axis.

1 mark for open/closed circles in correct place

1 mark for $x>1$ and $x<0$

(c) Calculate

$$P(X < \frac{2}{3} \mid X > \frac{1}{3})$$

3

$\frac{P(A \cap B)}{P(B)} = \frac{P(x < \frac{2}{3} \cap x > \frac{1}{3})}{P(x > \frac{1}{3})}$ 1mark for understanding & using conditional probability

$= \frac{\int_{\frac{1}{3}}^{\frac{2}{3}} 4x^3\,dx}{\int_{\frac{1}{3}}^{1} 4x^3\,dx}$

$= \frac{(\frac{2}{3})^4 - (\frac{1}{3})^4}{1^4 - (\frac{1}{3})^4}$ 1mark.

$= \frac{\frac{16}{81} - \frac{1}{81}}{1 - \frac{1}{81}}$

$= \frac{\frac{15}{81}}{\frac{80}{81}}$

$= \frac{15}{80}$ or $\frac{3}{16}$ 1 mark for correct answer.

OR $F(x) = \int_0^4 4x^3\,dx$

$= x^4$

$P(x > \frac{1}{3}) = 1 - F(\frac{1}{3})^4$

$= \frac{80}{81}$

$P(x < \frac{2}{3}) = F(\frac{2}{3})$

$= (\frac{2}{3})^4 = \frac{16}{81}$

$P(x < \frac{1}{3}) = (\frac{1}{3})^4 = \frac{1}{81}$

25. The population L in thousands of mountain lions, who are predators, and the population D in thousands of deer, who are the prey, in a particular continuously monitored region can be modeled by the following two periodic functions.

$$L = 9 + 6\sin\frac{\pi}{4}(t-1)$$

$$D = 18 + 9\cos\frac{\pi}{4}t$$

Where t is the time in years after the Tracking with Technology started, L and D are the populations (in thousands) of lions and deer respectively.

(a) How much time passes between successive maximum lions' populations (the period)? **2**

$L = 9 + 6\sin\frac{\pi}{4}(t-1)$

Period $= \frac{2\pi}{n}$

$= \frac{2\pi}{\frac{\pi}{4}}$ 1 mark

$= 8$ years 1 mark.

(b) Determine the initial populations of lions and deer respectively. **2**

When $t = 0$

$L = 9 + 6\sin\frac{\pi}{4}(-1)$ $D = 18 + 9\cos(\frac{\pi}{4} \times 0)$

$= 9 + 6 \times -\frac{1}{\sqrt{2}}$ $= 18 + 9$

$= 27$ thousand

$= 9 - 3\sqrt{2}$ $= 27000$

1 mark.

$\doteqdot 4.757$ thousand

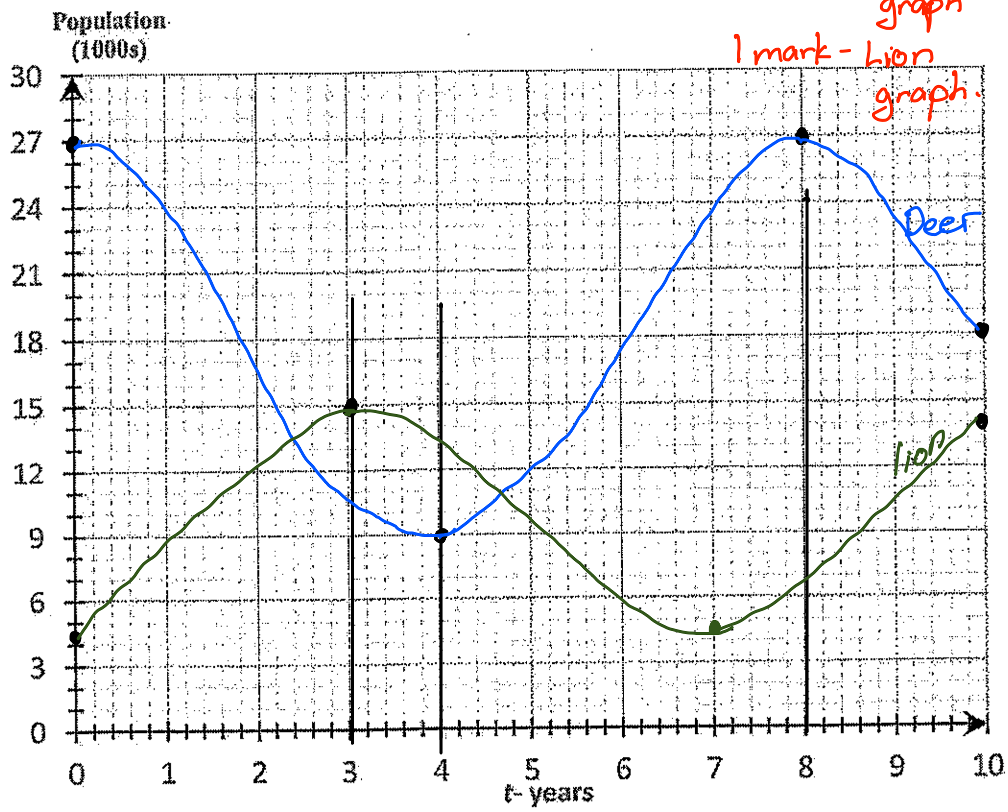
$= 4757$ lions 1mark

$\therefore$ Initially there were 4757 lions and 27000 deer.

(c) Sketch the graphs of the two functions (the Lion and Deer functions) on the same axes in the domain $0 \leqslant t \leqslant 10$ **2**

1 mark - Deer graph

1 mark - Lion graph.



(d) Over which time period, $0 \leqslant t \leqslant 10$, was the deer population increasing while the lion population decreased? **1**

$4 < t < 7$ from graph.

1 mark

must have a graph.

26. Solve $\ln x - \frac{6}{\ln x} = 5$ **2**

$(\ln x)^2 - 6 = 5\ln x$

$(\ln x)^2 - 5\ln x - 6 = 0$

Let $u = \ln x$

$u^2 - 5u - 6 = 0$ $1, \times -6, 1$ 1 mark

$(u-6)(u+1) = 0$

$u = 6$ or $u = -1$

$\therefore \ln x = 6$ or $\ln x = -1$

$x = e^6$ or $x = \frac{1}{e}$ 1 mark.

$T_1 \quad T_2 \quad T_3$

27. If $x^2 = (2a - x)(2b - x)$ show that $\frac{1}{a}, \frac{1}{x}, \frac{1}{b}$ are in arithmetic sequence. 3

$$x^2 = 4ab - 2ax - 2bx + x^2$$

$$4ab - 2ax - 2bx = 0$$

$$4ab - 2x(a+b) = 0$$

$$2x = \frac{4ab}{a+b}$$

$$x = \frac{2ab}{a+b}$$

$$\frac{1}{x} = \frac{a+b}{2ab}$$ 1 mark

$$\therefore T_2 - T_1 = \frac{a+b}{2ab} - \frac{1}{a}$$

$$= \frac{a+b-2b}{2ab}$$

$$= \frac{a-b}{2ab}$$ 1 mark

and $$T_3 - T_2 = \frac{1}{b} - \frac{a+b}{2ab}$$

$$= \frac{2a-(a+b)}{2ab}$$

$$= \frac{a-b}{2ab}$$

Since $T_2 - T_1 = T_3 - T_2 = \frac{a-b}{2ab}$ 1 mark for clear reason.

then $\frac{1}{a}, \frac{1}{x}, \frac{1}{b}$ are in arithmetic sequence.

End Section III

Q29 /6
Q 30,31 /13
Q 28,32 /11

Section IV (30 marks)
Answer in the space provided.

Student Number: ______________

28. Katie has deliberately designed a biased six sided die, with the following probability distribution for X, the number on the uppermost face when the die is rolled.

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1-\theta}{6}$	$\frac{1-\theta}{6}$	$\frac{1-\theta}{6}$	$\frac{1+\theta}{6}$	$\frac{1+\theta}{6}$	$\frac{1+\theta}{6}$

(a) What values of θ are feasible in order for $P(X)$ to be a probability function? **2**

$3\left(\frac{1-\theta}{6}\right) + 3\left(\frac{1+\theta}{6}\right) = 1$ (1)

$\frac{6 - 3\theta + 3\theta}{6} = 1$

$\frac{6}{6} = 1$ ∴ works for all θ

But $\frac{1-\theta}{6} \geq 0$ and $\frac{1+\theta}{6} \geq 0$

$\theta \leq 1$ and $\theta \geq -1$

∴ $-1 \leq \theta \leq 1$ (1)

(b) Find $P(0 \leq X \leq 4)$ **1**

$3\left(\frac{1-\theta}{6}\right) + \frac{1+\theta}{6}$

$= \frac{4-2\theta}{6} = \frac{2-\theta}{3}$ (1)

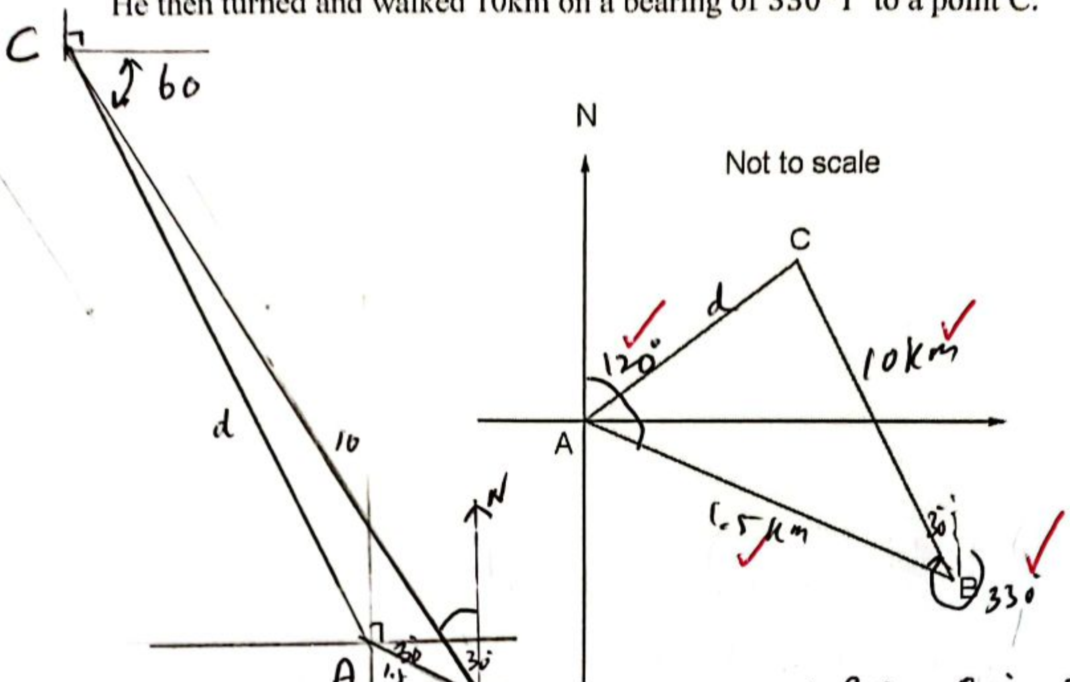
(c) Find the probability of rolling an even number? **1**

$P(2) + P(4) + P(6)$

$= \frac{1-\theta}{6} + \frac{2(1+\theta)}{6}$

$= \frac{3+\theta}{6} = \frac{1}{2} + \frac{\theta}{6}$ (1)

29. A student on a bike left point A and walked 1.5km on a bearing of 120^0T to a point B.
He then turned and walked 10km on a bearing of 330^0T to a point C.



any 2 ✓ 1m

$\angle CBA = 90° - 30° - 30 = 30$

(a) Label all features on the diagram above. **2**

(b) Calculate the distance from point C to point A. (give your answer to 2 decimal places) **2**

$d^2 = 10^2 + 1.5^2 - 2(10)(1.5)\cos 30°$

$d^2 = 102.25 - 30 \times \sqrt{3}/2$

$= 76.2692$

$d = 8.7332 \; (d > 0)$

$d = 8.73$ km (2dp)

(c) What is the bearing of A from C ? (Give your answer to the nearest degree) **2**

$1.5^2 = (8.7332)^2 + 10^2 - 2(8.7332)(10)\cos\angle ACB$

$\cos\angle ACB = \dfrac{176.2692 - 2.25}{174.664}$

$= 0.9963$

$\angle ACB = 4.9247° = 4°55' \doteq 5°$ (nearest degree)

Bearing of A from C $= 90° + 60° + 5° = 155°$ T

(nearest degree)

30. A particle is moving in a straight line. Initially, it is travelling to the left at 1cm/min. Its acceleration is given by:

$$a = \pi \cos\ (\pi t) + \pi \sin\ (\pi t)$$

for $0 \le t \le 2$ where time and displacement are measure in minutes and cm respectively.

(a) Find when the particle first changes its direction. **2**

$v = \sin(\pi t) - \cos(\pi t) + c$

$t = 0 \quad v = -1 \quad 0 - 1 + c = -1$

$\therefore c = 0$

$\therefore v = \sin(\pi t) - \cos(\pi t)$ (1)

change direction when $v = 0$

$\therefore \sin(\pi t) = \cos(\pi t)$

$\tan(\pi t) = 1$

$\pi t = \frac{\pi}{4}, \frac{5\pi}{4}$ etc (1)

1st change direction when $t = \frac{1}{4}$ min

(b) Find the total distance travelled in the first half a minute. **3**

$x = \left| \int_0^{\frac{1}{4}} (\sin \pi t - \cos \pi t)\,dt \right| + \int_{\frac{1}{4}}^{\frac{1}{2}} (\sin \pi t - \cos \pi t)\,dt$ (1m)

$x = \left| \left(\frac{-\cos \pi t}{\pi} - \frac{\sin \pi t}{\pi} \right) \Big|_0^{\frac{1}{4}} \right| + \left[\frac{-\cos \pi t}{\pi} - \frac{\sin \pi t}{\pi} \right]_{\frac{1}{4}}^{\frac{1}{2}}$

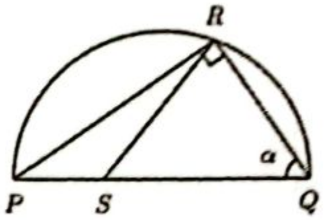
$= \left| \frac{1}{\pi} \left[-\cos\frac{\pi}{4} - \sin\frac{\pi}{4} \right) \right| + \frac{1}{\pi}(\cos 0 + \sin 0) - \left(\frac{\cos\frac{\pi}{2}}{\pi} + \frac{\sin\frac{\pi}{2}}{\pi} \right) + \frac{\cos\frac{\pi}{4}}{\pi} + \frac{\sin\frac{\pi}{4}}{\pi}$ (1m)

$= \left| \frac{1}{\pi}\left(-\frac{2}{\sqrt{2}} + 1 \right| - \frac{\pi}{\pi} + \frac{2}{\sqrt{2}} \right)$

$= \frac{1}{\pi}\left[(\sqrt{2} - 1) - 1 + \sqrt{2} \right] = 0.2637$ (4dp)

$= \frac{2\sqrt{2} - 2}{\pi}$ (1m)

31. ΔPQR is a right-angled triangle inscribed in a semi-circle. R is a variable point on the circumference. The point S lies on PQ such that $SQ = kQR$ where k is a positive constant. If $PQ = d\ cm$ and $\angle PQR = \alpha$ radians:



(a) Show that the area of ΔSQR is $A = \frac{1}{2}kd^2\cos^2\alpha\sin\alpha$. **3**

Area $\Delta SQR = \frac{1}{2} SQ . QR \sin\alpha$

$= \frac{1}{2}(kQR)(QR)\sin\alpha$ 1m

$= \frac{1}{2}k(QR)^2\sin\alpha$ 1m ($QR = d\cos\alpha$)

$= \frac{1}{2}k(d\cos\alpha)^2\sin\alpha$ 1m ($SQ = kd\cos\alpha$)

$= \frac{1}{2}kd^2\cos^2\alpha\sin\alpha$

(b) Show that $\frac{dA}{d\alpha} = \frac{1}{2}kd^2(3\cos^3\alpha - 2\cos\alpha)$. **2**

$\frac{dA}{d\alpha} = \frac{1}{2}kd^2\frac{d}{d\alpha}(\cos^2\alpha\sin\alpha)$

$= \frac{1}{2}kd^2[\cos^2\alpha\cos\alpha + (\sin\alpha)2\cos\alpha(\sin\alpha)]$ 1m

$= \frac{1}{2}kd^2(\cos^3\alpha - 2\cos\alpha(\sin^2\alpha))$

$= \frac{1}{2}kd^2(\cos^3\alpha - 2\cos\alpha(1-\cos^2\alpha))$ 1m

$= \frac{1}{2}kd^2(\cos^3\alpha - 2\cos\alpha + 2\cos^3\alpha)$

$= \frac{1}{2}kd^2(3\cos^3\alpha - 2\cos\alpha)$

(c) Find the greatest possible area of ΔSQR in terms of k and d. **3**

Greatest possible area when $\frac{1}{2}kd^2(3\cos^3\alpha - 2\cos\alpha) = 0$ — 1m

Since $k > 0$, $d > 0$ $\quad \cos\alpha(3\cos^2\alpha - 2) = 0$

$\therefore \cos\alpha = 0$ or $\cos^2\alpha = \frac{2}{3}$

$\cos\alpha = 0$ or $\cos\alpha = \pm\sqrt{\frac{2}{3}}$

Since $0 < \alpha < \frac{\pi}{2}$ $\therefore \cos\alpha = \sqrt{\frac{2}{3}}$ only

1m for $0 < \alpha < \frac{\pi}{2}$ $\therefore \cos\alpha \neq 0$, $\cos\alpha \neq -\sqrt{\frac{2}{3}}$

when $\cos\alpha = \sqrt{\frac{2}{3}}$, $\alpha = 0.61547$

Area $= \frac{1}{2}kd^2\cos^2\alpha\sin\alpha = \frac{1}{2}kd^2 \cdot \frac{2}{3}\sqrt{\frac{1}{3}}$

$= \frac{kd^2}{3\sqrt{3}}$

Check max/min

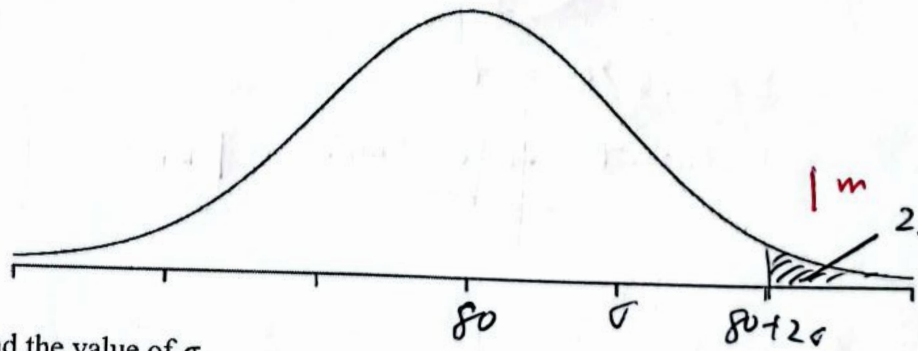
α	0.5	0.61547	1
A'	$0.2224 \times \frac{1}{2}kd^2$	0	$-0.6074 \times \frac{1}{2}kd^2$
	/	—	\

$\therefore$ max A at $\cos\alpha = \sqrt{\frac{2}{3}}$

1m for max A & check

32. An airline company operates regular flights between cities A and B. The flight time X is normally distributed with a mean of 80 minutes and standard deviation σ minutes. Also, 2.5% of the flights take longer than 96 minutes to arrive at the destination.

(a) Label the normal distribution curve below with the given information. **1**



many forgot 2.5%

(b) Find the value of σ. **1**

$80 + 2\sigma = 96$

$\sigma = 8$ (1m)

(c) What percentage of flights take longer than 90 minutes? **2**

$z = \frac{90-80}{8} = 1.25$ 1 m

10.56 % from z table

1 m

same answer as 0.1056 get 1 m

(d) In order to attract customers, the management of the airline company decides to refund the fares if a flight takes longer than $(80 + t)$ minutes. The company wants to keep 99.5% of the fares. **3**

0.5% = 0.005

z = 2.575 1 m

$z = \frac{x - \bar{x}}{\sigma}$

$2.575 = \frac{80 + t - 80}{80}$ 1 m

$8 \times 2.575 = t$

t = 20.6 min 1 m